PION 2023
Question Booklet

Natuurwetenschappelijke Studievereniging Amsterdam

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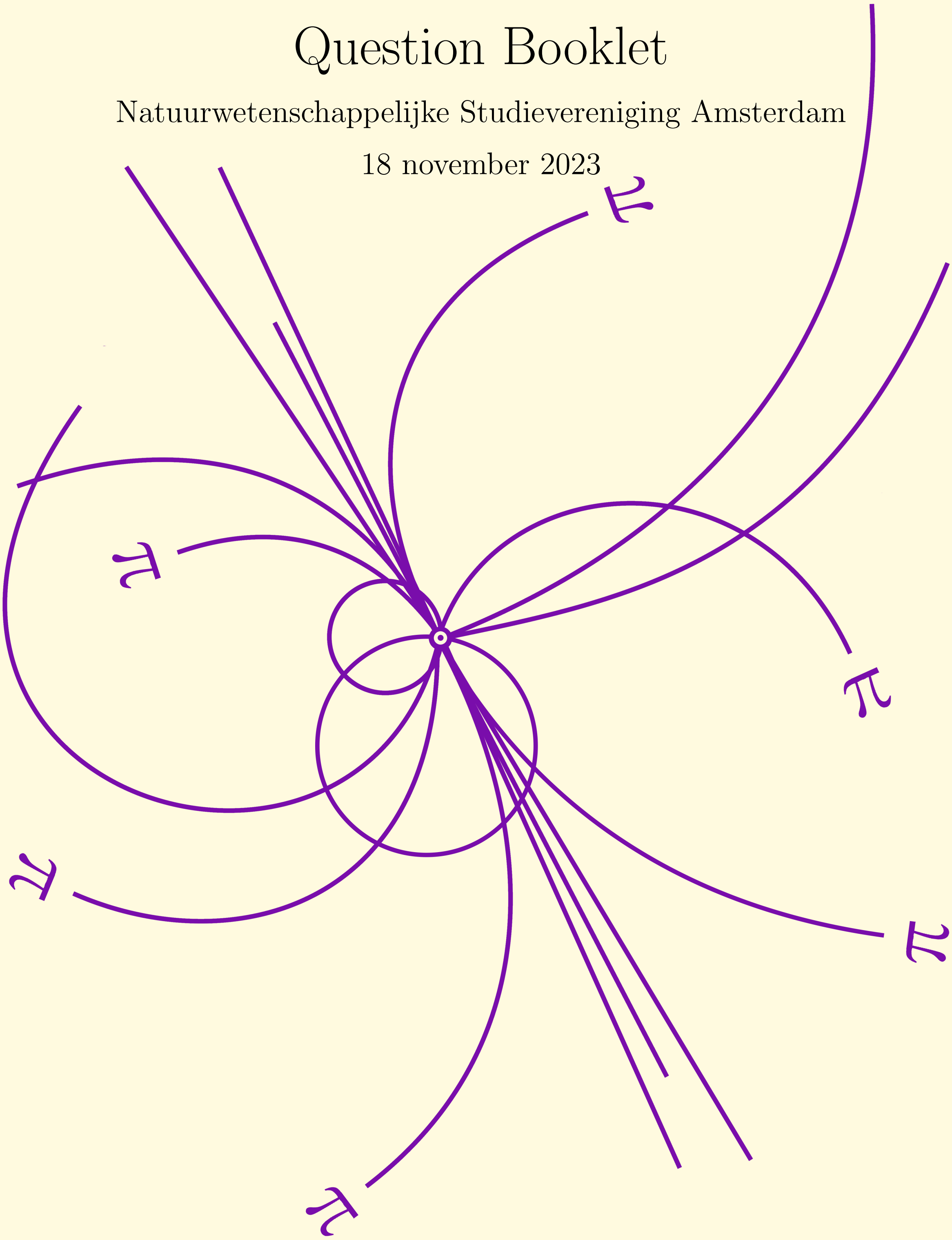


PION 2023

This exam starts at 13:00 and you have three hours to complete the most you can. This exam contains 7 questions, each of which is worth ten points. You can work together with your group on all questions. Please hand in each question once, on a separate sheet, numbered with the corresponding exercise. We wish you the best of luck!

Question 1: Simple Model System for Rod-Like Particles

J. de Graaf

October 5, 2023

We consider a system of long colloidal rods suspended in a molecular fluid. A simple model of of this system views the rods as rectangular blocks of length L and thickness D, i.e., of the form $L \times D \times D$. A further simplification is to restrict the number of possible orientations of each rod to three, such that the main axes of the rods can only point in the direction of a laboratory frame $\hat{\boldsymbol{x}}_\alpha, \alpha = 1, 2, 3$. A particle with orientation α has its long axis along $\hat{\mathbf{x}}_\alpha$ and its minor axes aligned with the remaining axes. The rods are not confined to a lattice, so that their centers of mass can move throughout the volume V. to which they are contained. The interaction between the particles is hard, i.e., overlap is not allowed. We write ρ for the total number density, so that the total number of rod particles is given by $N = \rho V$. The Helmholtz free energy F of such rods at temperature T is given, within the second virial approximation, by

$$\frac{F}{V k_{\mathrm{B}} T} = \sum_{\alpha=1}^{3} \rho_\alpha \left(\log \rho_\alpha \mathcal{V} - 1\right) + \sum_{\alpha=1}^{3} \sum_{\alpha'=1}^{3} B_{\alpha\alpha'} \rho_\alpha \rho_{\alpha'}, \tag{1.1}$$

with ρ_α the density of particles with orientation α, and $\mathcal{V}$ the (irrelevant) thermal volume. The definition of the virial coefficient is given by

$$B_{\alpha\alpha'}(T) = -\frac{1}{2} \int_{\mathcal{V}} \left(\exp\left[-\beta \phi_{\alpha\alpha'}(r)\right] - 1\right) \mathrm{d}\mathbf{r}\,,$$

where the integrand is often referred to as the Mayer function. The pairwise interaction between rods of orientation α and α' is given by $\phi_{\alpha\alpha'}(\mathbf{r})$, where $\mathbf{r}$ is their center-to-center distance vector.

(a) (0.5 points) Argue that $B_{\alpha\alpha'}$ is a symmetric 3×3 matrix.

Define the excluded volume to be that region round the center of the particle with orientation α, where the center of the particle with orientation α' is not allowed to be, due to the hard-particle interaction.

(b) (0.5 points) Sketch the two possible configurations that can occur in our simple model. Next use your sketch to indicate the excluded volume.

(c) (0.5 points) Use the intuition that you developed in (b) to calculate the second virial coefficients $B_{11} = B_{22} = B_{33} \equiv B_{\|}$ and $B_{12} = B_{13} = B_{23} \equiv B_{\perp}$ for pairs of parallel and perpendicular rods, respectively.

From here on out, we will consider the "needle" limit $L/D \to \infty$, where we will take the leading order scaling to determine the behavior of the system.

(d) (1 point) First calculate $B_{\|}/L^2 D$ and $B_{\perp}/L^2 D$ in this limit, and then show that the dimensionless free energy $\psi = F L^2 D / V k_B T$ takes the form

$$\psi = \sum_{\alpha} c_\alpha (\log c_\alpha - 1 + \log \frac{\mathcal{V}}{L^2 D}) + 2(c_1 c_2 + c_1 c_3 + c_2 c_3), \tag{1.2}$$

with dimensionless densities $c_\alpha = L^2 D \rho_\alpha$.

The constant term $\log \mathcal{V}/L^2 D$ can be ignored; it is an irrelevant offset of the free energy of chemical potential. Define the *nematic order parameter* S by $c_3 = \frac{1+2S}{3} c$ and $c_1 = c_2 = \frac{1-S}{3} c$, with $c = c_1 + c_2 + c_3 = \rho L^2 D$ the total dimensionless density. This definition of S selects the $\hat{\boldsymbol{x}}_3$ axis as special, which is simply a labeling convention.

(e) (0.5 points) Sketch the behavior the this system at very low (ideal-gas-like) dilution.

(f) (0.5 points) Increasing the dilution slightly, we anticipate a phase transition. Argue which degree of continuous freedom will become discrete first. Illustrate your answer by a sketch.

(g) (0.5 points) Use your result from (e) to explain the nomenclature of "nematic order parameter".

(h) (1 point) Give the range of the parameter S, keeping in mind that densities are non-negative.

(i) (0.5 points) Show that

$$\psi(c,S) = c\left(\frac{2}{3}c(1-S^2)-1\right) + \frac{1}{3}c\log\left[\frac{c^3}{27}(1-S)^2(1+2S)\right] - \frac{2}{3}cS\log\left[\frac{1-S}{1+2S}\right].$$

For a given c one needs to determine S such that it minimizes ψ (at the fixed c).

(j) (0.5 points) Show that $S=0$ is a solution of $(\partial\psi/\partial S)_{S=0}=0$ for any c. Which phase is associated with the solution $S=0$?

(k) (1 point) The result of (j) does *not* guarantee that $S=0$ yields a minimum of ψ. Argue on the basis of $(\partial^2\psi/\partial S^2)_{S=0}$ that ψ is minimized by $S\neq 0$ at $c>c^*$, and calculate c^*. Which phase do you associate with $S\neq 0$?

(l) (2 points) Phase coexistence of a low-density isotropic phase, with density c_I and order parameter $S_I=0$, and a high-density nematic phase, with density c_N and order parameter S_N, requires *three* conditions to fix the three unknowns c_I, c_N, and S_N. Give these conditions.

The coexistence conditions involve nonlinear algebraic equations that can easily be determined numerically. One finds that $c_I = 1.258$, $c_N = 1.915$, and $S_N = 0.915$.

(m) (0.5 points) Provide the conditions for which the systems is (metastable) isotropic, nematic, and phase separated, the above c_I, c_N and c^*?

(n) (0.5 points) Estimate, for hard rods with $L/D = 100$, the packing fractions beyond which orientational ordering is to be expected on the basis of the above result. The packing fraction is the number density times the volume of a particle.

Question 2:
“Oppenheimer” - Igniting the atmosphere

M. Beyer

October 1, 2023

During the Manhattan Project, scientists grappled with the potential of igniting the Earth’s atmosphere through nuclear explosions, specifically focusing on the idea of nuclear fusion, which could lead to the creation of a hydrogen bomb. A 1959 interview with Arthur Compton, a Manhattan Project leader, vividly depicts the dramatic concerns. Compton was alarmed by the possibility that an atomic explosion might trigger a chain reaction, potentially causing a catastrophic explosion, including the vaporization of Earth. He worried about the instability of nitrogen in the atmosphere.

In order for the nuclear reaction to be sustained, it must produce at least as much energy as is lost by other processes, otherwise the temperature will drop and the reaction comes to an end.
The Rydberg energy is given by

$$E = -\frac{hc\mathcal{R}Z^2}{n^2}, \tag{2.1}$$

where $hc\mathcal{R} = 2.179 \times 10^{-18}$ J. You can also use:
$1eV = 1.6 \cdot 10^{-19} J$
$m_p = 1.007 amu$
$m_n = 1.008 amu$
$1 amu = 1.66 \cdot 10^{-27}$kg. There is a periodic table at the end of the booklet.

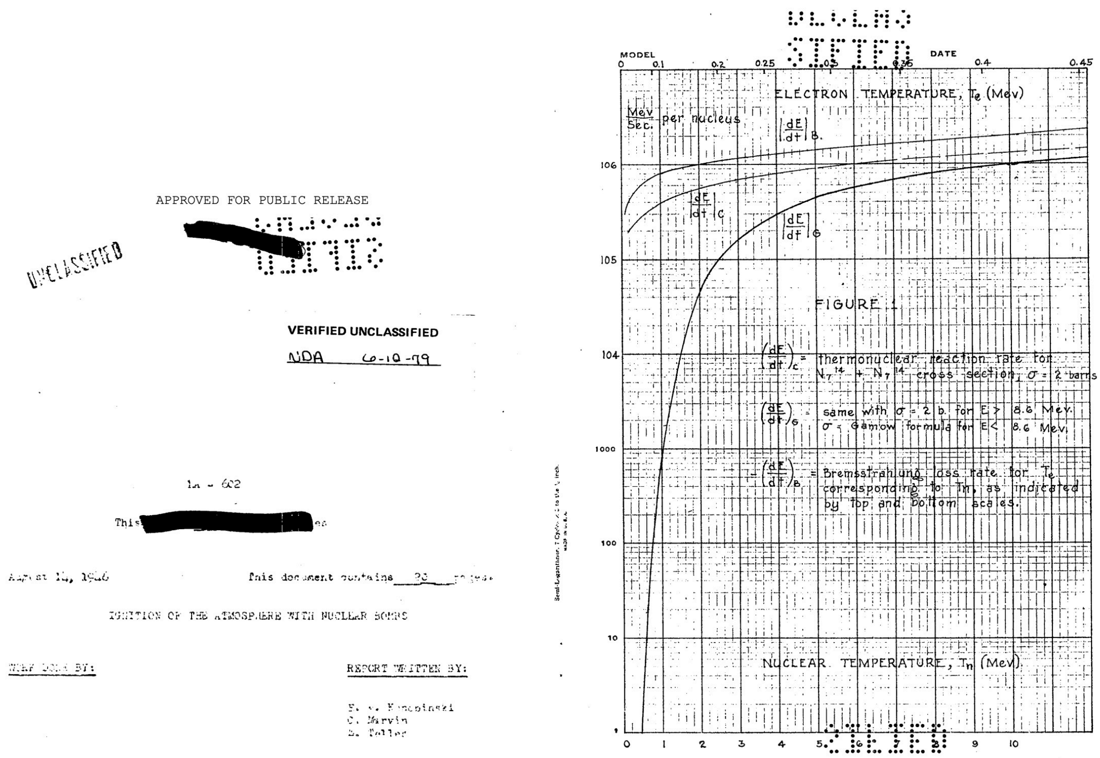


Figure 1: E. J. Konopinski, C. Marvin, and E. Teller, “Ignition of the Atmosphere with Nuclear Bombs”, Los Alamos National Laboratory, LA-602, April 1946.

(a) (1 point) Complete the following nuclear reactions that could take place in the atmosphere:

$$^{14}_{7}N \ + \ ^{14}_{?}N \rightarrow ^{24}_{12}? \ + \ ? \tag{2.2}$$
$$^{14}_{7}N \ + \ ^{14}_{?}N \rightarrow ^{16}_{8}? \ + \ ? \tag{2.3}$$

Show your working out and discuss conservation laws of the mass (A) and atomic (Z) number.

(b) (1 point) Calculate the energy in mega electronvolts (MeV) that is released in each reaction.

For nuclear fusion to happen, the nuclei involved must first overcome the electric repulsion to get close enough for the attractive nuclear strong force to take over to fuse the nuclei.

(c) (2 points) Sketch an energy diagram along the reaction coordinate (i.e., the distance between the nuclei) for reaction (1) and indicate all barriers that need to be overcome.
Hint: It might help to consider the nuclear reaction in forward and backward reaction.

Calculate the barrier height(s) in MeV, assuming that the radius of the nuclei is given by $r = A^{1/3} \times 1.2 \times 10^{-15}$m. Discuss your results in relation to what was found in part (b).

Can you imagine why nuclear reactions can take place at energies that lie below the barrier height(s)? How could this make reaction (2.3) more efficient than reaction (2.2)?

The most important cooling mechanisms we have to consider is bremsstrahlung. At the very high temperatures in a fusion weapon, the atoms are broken up into nuclei and electrons, forming a plasma. In a nuclear reaction, the resulting energy is primarily used to heat atomic nuclei. These need not be in thermal equilibrium with the electrons, therefore we speak of a nuclear temperature. But the nuclei give some of their energy to electrons by collisions, and thus heat the electrons. The electrons, in turn, lose their energy to radiation (bremsstrahlung).

(d) (2 points) Estimate the energy it needs to rip off the last electron of a nitrogen atom, so that it is then fully ionized. How likely is this at a nuclear temperatures of a few MeV, as indicated on the horizontal axis in figure 1?

(e) (1 point) For bremsstrahlung the radiated power is proportional to the second time derivative of a dipole moment $\vec{d} = q\vec{r}$. Explain why collisions of like particles (electron-electron) don't lead to bremsstrahlung. Hint: Consider the dipole moment for two colliding particles and the center-of-mass.

In electron-ion bremsstrahlung, which particles are the main radiators?

(f) (2 points) Another important cooling mechanism is the inverse Compton effect, in which radiation picks up energy from fast electrons. Normal Compton scattering is commonly described as inelastic scattering, because the energy of the scattered photon E_2 is less than the energy of the incident photon E_1. The electron is assumed to be initially at rest (so we only need to take into account the rest mass $E_0 = m_e c^2$) and after the collision the energy is $(E_0^2 + p_e c^2)^{1/2}$, with the electron momentum p_e.

Derive the Compton equation

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - cos(\theta)), \tag{2.4}$$

with the scattering angle θ and the photon wavelengths before λ_1 and after λ_2 Compton scattering. At what angle can the most efficient energy transfer be observed?

(g) (1 point) The resulting rates for energy production $(dE/dt)_G$ assuming the geometric cross section for reaction (2.2) and energy loss through bremsstrahlung $(dE/dt)_B$ are depicted in Figure 1. Can a nuclear bomb ignite the atmosphere?

Question 3:
Falling Slinky

M. van Exter

October 1, 2023

Consider a slinky (a flexible, open spring) suspended from its top and at rest. When you release the top end, the time evolution of the slinky is fascinating, as shown in the series of pictures below. To describe this phenomenon, we consider an ideal uniform slinky of mass m, zero pretension, and negligible rest length, for which each segment obeys Hooke's law: $F = kL$

Figure 2: The behaviour of a slinky dropped from being held at the top. Notice the bottom of the slinky does not move until the top has "caught up".

(a) (2.5 points) Describe the (vertical) shape of the slinky at rest(left frame).
Hint: Denote points on the slinky by a dimensionless variable x, ranging from $x = 0$ at the bottom to $x = l$ at the top and describe its shape by specifying the height $L(x)$ of each segment above the bottom of the slinky.

(b) (2.5 points) Explain in words why the slinky behaves the way it does while falling

(c) (2.5 points) How long will it take for the top of the slinky to reach the bottom of the slinky? How does this result compare with the fall time of a small object falling from the same height L?

(d) (2.5 points) Derive equations to describe the distance $\Delta L(t)$ travelled by the top of the slinky at a time t after 'launch', up to the moment when it reaches the bottom of the slinky.

Question 4: Charged particles around a black hole

G. Koekoek

October 5, 2023

In this exercise, the unit system $c = G = 1$ is used. All answers can be expressed in these units.

The spherically symmetric vacuum solution of the Einstein Field Equations,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}, \tag{4.1}$$

is given by the famous Schwarzschild spacetime: a non-rotating black hole of mass M. If this spacetime is doused in a constant magnetic field F^r_φ that is small enough to have negligible effect on the curvature of spacetime, equatorial and circular motion of a charged particle of mass m and charge q is given by:

$$\frac{M}{r^2}\left(1 - \frac{2M}{r}\right)(u^t)^2 - r\left(1 - \frac{2M}{r}\right)(u^\varphi)^2 = \frac{q}{m} F^r_\varphi u^\varphi. \tag{4.2}$$

Here, $u^\mu = (u^t, u^r, u^\theta, u^\varphi)$ is the 4-velocity of the particle, measured in proper time τ. All these velocities are constant. The only non-zero component magnetic field $\mathbf{B}$ corresponding to F^r_φ is perpendicular to the equatorial plane, using that in this geometry

$$|F^r_\varphi| = |B_\theta| = |B_z| r \left(1 - \frac{2M}{r}\right). \tag{4.3}$$

(a) (2 points) Explain, without calculations, for each of these velocities $(u^t, u^r, u^\theta, u^\varphi)$, why it is constant.

This particle does not simply follow the usual (Newtonian) Kepler's Third Law, but a modified one based on the fact that it moves in a gravitational field and is subject to a magnetic field.

(b) (2 points) Show that the modified version of Kepler's Third Law is given by

$$u^\varphi = \frac{-\frac{q}{m}F^r_\varphi \pm \sqrt{\left(\frac{q}{m}F^r_\varphi\right)^2 + \frac{4M}{R}\left(1 - \frac{3m}{R}\right)}}{2R\left(1 - \frac{3m}{R}\right)}, \tag{4.4}$$

in which R is the radius of the circular orbit.

In absence of a magnetic field, the smallest allowed circular orbit around a Schwarzschild black hole has radius $R = 3M$. From the result of exercise b, we see that circular motions exist around the black hole with smaller radii, provided the magnetic field is large enough. A magnetic dipole field F^r_φ with magnetic dipole moment μ in a Schwarzschild spacetime, can be shown to be given by:

$$F^r_\varphi = \frac{\mu}{R^2}\left(1 - \frac{2M}{r}\right)\left(h - R\frac{\partial h}{\partial R}\right), \tag{4.5}$$

where

$$h(R) = \frac{3R^3}{8M^3}\left(\ln\left(1 - \frac{2M}{R}\right) + \frac{2M}{R} + \frac{2M^2}{R^2}\right). \tag{4.6}$$

This results in some interesting regions of allowed circular orbits R. For a range of values $\mu > 0$, there exists a region $2M < R < 3M$ in which no orbits are allowed: a forbidden zone. However, if μ is made big enough, circular orbits are allowed for all values R in between $2M < R < 3M$, and the forbidden zone disappears.

(c) (2 points) From the expression in part (b), calculate the formula for the non-zero magnetic field strength μ needed to make the forbidden zone disappear.

While the particle is doing its orbiting, it will send out both electromagnetic radiation and gravitational waves, losing energy to both in the process. In what follows, we will investigate the stability of these orbits under this energy loss. To do so, we will need the energy loss due to electromagnetic radiation. Maxwell's electrodynamics teaches us that the power P_{elec} sent out by an accelerating particle in the non-curved background of Minkowski spacetime is given by

$$P_{\text{elec}} = \frac{q^2\gamma^4}{6\pi}\left(\vec{a}^2 + \gamma^2(\vec{v}\cdot\vec{a})^2\right), \tag{4.7}$$

where γ is the special relativistic Lorentz-factor, $\vec{v}$ is the particle's orbital velocity, and $\vec{a}$ its corresponding acceleration, both measured in t.

(d) (2 points) Using Newton's Second Law $\frac{\mathrm{d}p}{\mathrm{d}t} = F$ in a Minkowski background and the Minkowski line-element, show that in out current situation the following hold:

$$|\vec{a}| = \frac{q|\vec{v}|}{\gamma^2 m^2}|B_z|, \tag{4.8}$$

$$\vec{a}\cdot\vec{v} = 0. \tag{4.9}$$

Here, the inner product is the usual three-dimensional dot-product.

Using the result of exercise d, we can calculate the electromagnetic energy loss in a Minkowski background. The power sent out by gravitational waves for a mass m in a circular orbit around a mass M and m is, in a Minkowski background, given by the Peters-Mathews equation:

$$P_{GW} = \frac{32}{5}(M_c\omega)^{10/3} \qquad \text{in which} \qquad M_c = \frac{(mM)^{3/5}}{(m+M)^{1/5}}. \tag{4.10}$$

Taking the ratio of P_{elec} and P_{GW}, we can now calculate which of the two radiations dominates the energy loss. However, the formulas for the powers studied in c and d work in a Minkowski background. In our current sutiation we need a Schwarzschild background.

(e) (2 points) Explain how the ratio between electromagnetic power and gravitational wave power changes when we move from Minkowski background to Schwarzschild background. *Note: no calculations necessary.*

Question 5:
Supersymmetric quantum mechanics and exactly solvable models

M.V. Mostovoy

November 4, 2023

Consider one-dimensional motion of a particle described by the Schrödinger equation,

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x)\right)\psi(x) = E\psi(x). \tag{5.1}$$

It is convenient to measure distances in units of the length scale of U, a, and energy in units of $\frac{\hbar^2}{2ma^2}$, which gives

$$\left(-\frac{d^2}{dx^2} + V(x)\right)\psi(x) = \epsilon\psi(x), \tag{5.2}$$

where $V = \frac{2ma^2 U}{\hbar^2}$ and $\epsilon = \frac{2ma^2 E}{\hbar^2}$. In what follows, $-\frac{d^2}{dx^2} + V(x)$ is called Hamiltonian and ϵ is called energy. We shall discuss a way to find $V(x)$, for which the eigenstates of bound states can be found exactly.

(a) (1 point) Consider two Hamiltonians, $\hat{H}_1 = \hat{A}^\dagger\hat{A}$ and $\hat{H}_2 = \hat{A}\hat{A}^\dagger$, where

$$\hat{A} = -\frac{d}{dx} + W(x) \quad \text{and} \quad \hat{A}^\dagger = \frac{d}{dx} + W(x) \tag{5.3}$$

with real $W(x)$. Show that $\hat{H}_1$ and $\hat{H}_2$ are Hermitian operators with non-negative eigenvalues.

(b) (1 point) Prove that if $\hat{H}_1$ has an eigenfunction $\psi_1(x)$ with a non-zero energy ϵ, then there is an eigenfunction $\psi_2(x)$ of $\hat{H}_2$ with the same energy. Find a relation between $\psi_1(x)$ and $\psi_2(x)$.

(c) (2 points) Prove that the wave function $\psi_1(x)$ of the zero-energy state of $\hat{H}_1$ (if it exists) satisfies

$$\hat{A}\psi_1(x) = 0, \tag{5.4}$$

whereas an equation for the wave function $\psi_2(x)$ of the zero-energy state of $\hat{H}_2$ is

$$\hat{A}^\dagger\psi_2(x) = 0. \tag{5.5}$$

It is easy to solve equations (5.4) and (5.5) for arbitrary $W(x)$ and show that only one of the two Hamiltonians (or none) can have the zero-energy eigenstate.

(d) (2 points) Show that for $W(x) = N\tanh(x)$,

$$V_1(x) = N^2 - \frac{N(N-1)}{\cosh^2 x} \quad \text{and} \quad V_2(x) = N^2 - \frac{N(N+1)}{\cosh^2 x}. \tag{5.6}$$

(e) (2 points) For $N = 1$, $V_1(x) = 1$. Find the ground state energy ϵ and the corresponding eigenfunction $\psi_1(x)$ of $\hat{H}_1$. Find the eigenfunction $\psi_2(x)$ of $\hat{H}_2$ with the same energy. Argue that $\psi_2(x)$ is not the ground state wave function.

(f) (2 points) Show that the ground state energy of $\hat{H}_2$ is 0 and find the corresponding wave function. Explain why $\hat{H}_2$ has no other bound states.

Hint: Solve equation (5.5) *for* $W(x) = \tanh(x)$.

To summarize, the Hamiltonian

$$\hat{H} = -\frac{d^2}{dx^2} - \frac{2}{\cosh^2 x} \tag{5.7}$$

has one bound state with the energy $\epsilon = -1$, the wave function of which can be found analytically. Considering $N = 2, 3, \ldots$, one can find in a similar way the wave functions and energies of all bound states of the Hamiltonian $\hat{H} = -\frac{d^2}{dx^2} - \frac{N(N+1)}{\cosh^2 x}$, but we stop here.

Question 6:
Measurement of the B meson decay time distribution at the PEP-II collider

W. Hulsbergen

An e^+e^- collider is a collider that collides electrons (e^-) with positrons (e^+). If the beam energies are chosen such that the total energy is around 10 GeV (about 10 times the proton mass), several resonances can be seen (Fig. 3). These resonances are called the Upsilon (Υ) resonances. They are meta-stable states $b\bar{b}$ states, where b is the bottom (or 'beauty') quark.

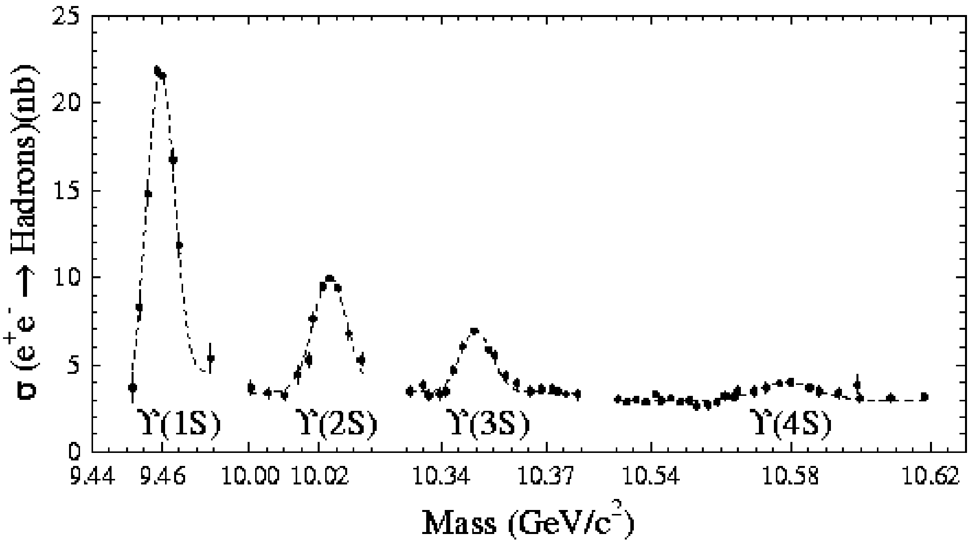


Figure 3: Particle production as a function of centre-of-momentum energy in the region of the Upsilon states as measured by the CUSB detector at Cornell in 1980.

Beauty mesons are mesons consisting of a beauty quark and a lighter anti-quark (or vice versa). The quark content of the four lightest beauty mesons is

$$B^0 = d\bar{b} \qquad \bar{B}^0 = \bar{d}b \qquad B^+ = u\bar{b} \qquad B^- = \bar{u}b$$

The rest mass of the B^0 and B^+ meson are almost identical, approximately $m_B = 5.279\ \mathrm{GeV}/c^2$. The $\Upsilon(4S)$ resonance at $M_\Upsilon = 10.580\ \mathrm{GeV}/c^2$ (the right-most bump in the figure) is just heavy enough for the decay into two beauty mesons: The majority of events at this collision energy is either $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$ or $e^+e^- \to \Upsilon(4S) \to B^+B^-$.

The decay of the upsilon to the two B mesons is a two-body decay: In the upsilon rest frame (also called the centre-of-momentum-system, or 'cms'), the two B mesons fly in opposite direction with the same momentum.

(a) (1 point) Compute the momentum p_{cms} of a B meson in the upsilon rest frame. Express your answer in M_Υ and m_B.
(Here, and in the remainder of the exercise, you may choose to work in natural units, such that $c = 1$.)

The PEP-II collider in Stanford is an e^+e^- collider tuned at the $\Upsilon(4S)$ resonance. PEP-II is an *asymmetric* collider: the positron beam has a lower energy than the electron beam, such that the two B mesons are boosted. The B mesons fly with almost the same velocity parallel to the electron beam.

(b) (1 point) Given that the positron beam has an energy of $E_+ = 3.1$ GeV, compute the energy E_- of the electron beam. You may ignore the electron and positron mass, e.g. assume $p(e^+) = E_+/c$ and $p(e^-) = E_-/c$. Express the answer in terms of E_+ and M_Υ.

(c) (1 point) Compute the momentum p_{lab} of a B mesons in the laboratory frame *ignoring* its momentum in the centre-of-momentum frame: That is, assume that the B particle is at rest in the $\Upsilon(4S)$ rest frame. Express your answer in M_Υ, m_B, E_- and E_+.
Hint: Remember that $p = \gamma\beta mc$, where $\gamma\beta$ is the boost factor. Compute the boost factor $\gamma\beta$ for the upsilon. If you ignore the velocity of the B in the upsilon frame, the boost factor for the B mesons is identical to that for the upsilon.

The decay time-distribution of an unstable particle usually follows an exponential law

$$N(t) \;=\; N_0\, e^{-t/\tau} \tag{6.1}$$

where τ is the mean decay time. B mesons have an average lifetime τ_B of about 1.5 ps. Due to a phenomenon called CP-violation there exists decays for which the decay time distribution of $B^0 \to X$ is different from the decay time distribution of $\bar{B}^0 \to \bar{X}$. The aim of the experiment at Stanford is to measure this small difference. Therefore, it is important to measure the decay times very precisely.

The decay length L_{lab} is the distance between the point of decay and the point of production of the B meson in the laboratory frame. The decay time in the laboratory is computed by dividing the decay length by the measured velocity:

$$t_{\text{lab}} \;=\; \frac{L_{\text{lab}}}{v_{\text{lab}}} \tag{6.2}$$

(d) (2 points) Show that the *proper* decay time t (e.g. the decay time in the rest frame of the B meson) can be computed as

$$t \;=\; \frac{L\, m_B}{p} \tag{6.3}$$

where m is the B meson rest mass, and p and L are respectively the B momentum and decay length in the *laboratory frame*, or any other frame in which the B meson is not at rest.

(e) (1.5 points) Compute the average B meson decaylength (the distance a B meson travels before it decays) in the cms frame. Express the result in the average proper time τ_B, m_B and your answer to exercise *a*.

(f) (1.5 points) Compute the average B meson decaylength in the laboratory frame, ignoring the velocity of the B meson in the e^+e^- rest frame. Express the result in τ_B, m_B and your answer to exercise *c*.

It is technologically easier to build a symmetric-energy collider (e.g. with $E_+ = E_-$) than an asymmetric-energy collider. Yet, it was chosen to use the latter strategy for the PEP-II collider.

(g) (2 points) The decay time resolution is determined by the decay length resolution. The latter is limited by technology: Typical particle detectors can reach a precision of about $\sigma(L) = 100\ \mu m$. Explain why the PEP-II collider was built as an asymmetric collider.

Question 7:
Light absorption by a photo-system followed by charge separation

I van Stokkum

A photo-system is made up of around 100 pigments that are coupled with each other, in order for the pigments to quickly distribute the excitation energy after absorption.
6% of the pigments (in the reaction centre) are able, by charge separation, to convert the excitation irreversibly to a chemically different state. The remaining 94% of the pigments are called the antenna. This way, light is converted in to chemical energy, which can be used to trap CO_2. Assume all pigments have equal energy.

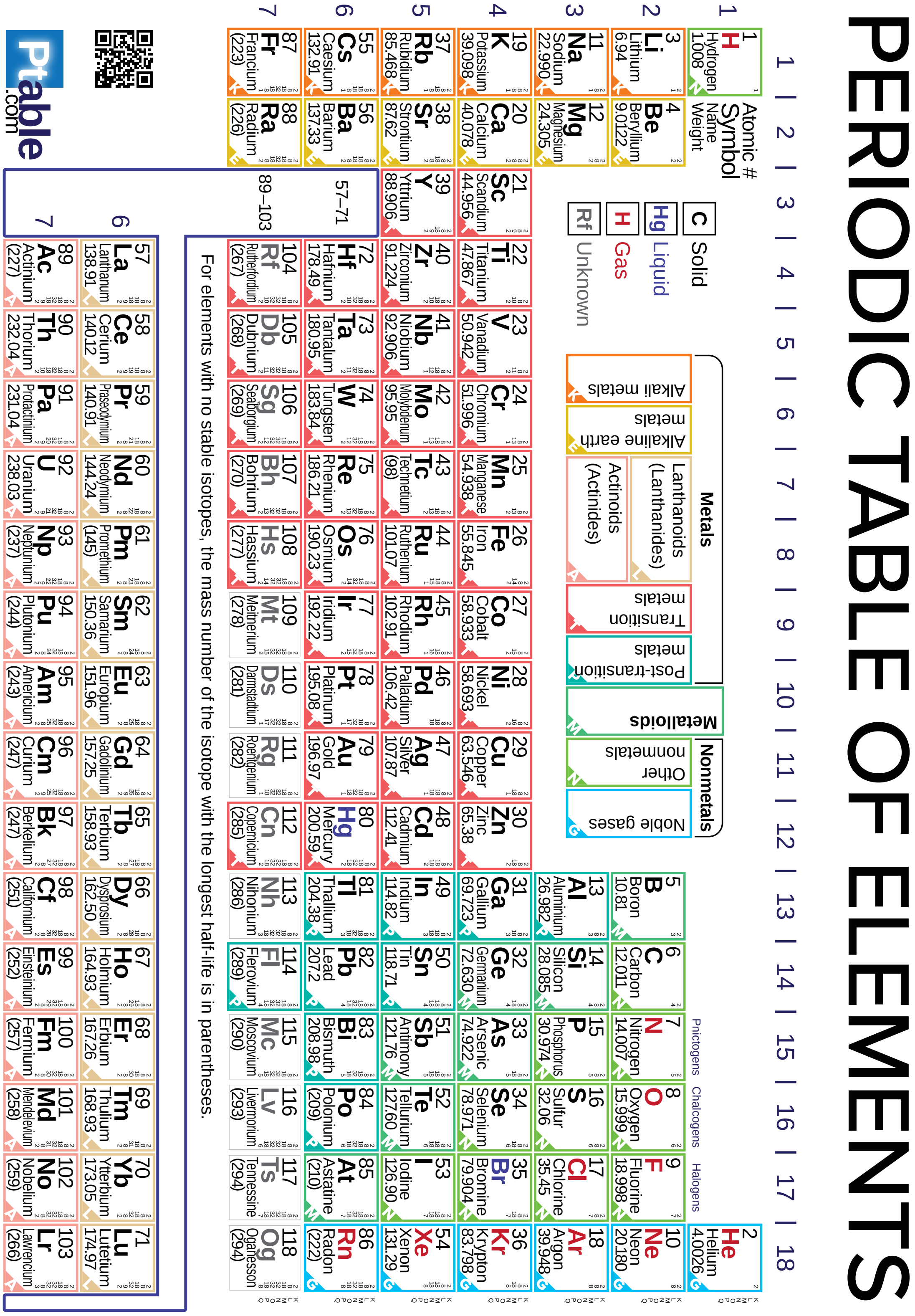
(a) (2 points) What percentage of excitations take place in the reaction centre?

(b) (2 points) What percentage of excitations take place in the antenna?

(c) (2 points) Assume an excited state has a lifetime of 1 nanosecond. Assume that charge separation has a speed of 1 picosecond. (The decay rate is the inverse of the lifetime.) What are the corresponding decay rates?

(d) (2 points) What percentage of absorbed photons are converted by charge separation into a chemically different state? Assume that the speed of energy transfer between antenna and the reaction centre are much larger than the natural decay rate, and can therefore be ignored.

(e) (2 points) How does nature make sure that the charge separation is irreversible?


PERIODIC TABLE OF ELEMENTS
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
1 2 3 4 5 6 7
Atomic # Symbol Name Weight
C Solid
Hg Liquid
H Gas
Rf Unknown
Metals
Alkali metals
Alkaline earth metals
Lanthanoids (Lanthanides)
Actinoids (Actinides)
Transition metals
Post-transition metals
Metalloids
Nonmetals
Other nonmetals
Noble gases
Pnictogens
Chalcogens
Halogens
1 H Hydrogen 1.008
2 He Helium 4.0026
3 Li Lithium 6.94
4 Be Beryllium 9.0122
5 B Boron 10.81
6 C Carbon 12.011
7 N Nitrogen 14.007
8 O Oxygen 15.999
9 F Fluorine 18.998
10 Ne Neon 20.180
11 Na Sodium 22.990
12 Mg Magnesium 24.305
13 Al Aluminium 26.982
14 Si Silicon 28.085
15 P Phosphorus 30.974
16 S Sulfur 32.06
17 Cl Chlorine 35.45
18 Ar Argon 39.948
19 K Potassium 39.098
20 Ca Calcium 40.078
21 Sc Scandium 44.956
22 Ti Titanium 47.867
23 V Vanadium 50.942
24 Cr Chromium 51.996
25 Mn Manganese 54.938
26 Fe Iron 55.845
27 Co Cobalt 58.933
28 Ni Nickel 58.693
29 Cu Copper 63.546
30 Zn Zinc 65.38
31 Ga Gallium 69.723
32 Ge Germanium 72.630
33 As Arsenic 74.922
34 Se Selenium 78.971
35 Br Bromine 79.904
36 Kr Krypton 83.798
37 Rb Rubidium 85.468
38 Sr Strontium 87.62
39 Y Yttrium 88.906
40 Zr Zirconium 91.224
41 Nb Niobium 92.906
42 Mo Molybdenum 95.95
43 Tc Technetium (98)
44 Ru Ruthenium 101.07
45 Rh Rhodium 102.91
46 Pd Palladium 106.42
47 Ag Silver 107.87
48 Cd Cadmium 112.41
49 In Indium 114.82
50 Sn Tin 118.71
51 Sb Antimony 121.76
52 Te Tellurium 127.60
53 I Iodine 126.90
54 Xe Xenon 131.29
55 Cs Caesium 132.91
56 Ba Barium 137.33
57–71
72 Hf Hafnium 178.49
73 Ta Tantalum 180.95
74 W Tungsten 183.84
75 Re Rhenium 186.21
76 Os Osmium 190.23
77 Ir Iridium 192.22
78 Pt Platinum 195.08
79 Au Gold 196.97
80 Hg Mercury 200.59
81 Tl Thallium 204.38
82 Pb Lead 207.2
83 Bi Bismuth 208.98
84 Po Polonium (209)
85 At Astatine (210)
86 Rn Radon (222)
87 Fr Francium (223)
88 Ra Radium (226)
89–103
104 Rf Rutherfordium (267)
105 Db Dubnium (268)
106 Sg Seaborgium (269)
107 Bh Bohrium (270)
108 Hs Hassium (277)
109 Mt Meitnerium (278)
110 Ds Darmstadtium (281)
111 Rg Roentgenium (282)
112 Cn Copernicium (285)
113 Nh Nihonium (286)
114 Fl Flerovium (289)
115 Mc Moscovium (290)
116 Lv Livermorium (293)
117 Ts Tennessine (294)
118 Og Oganesson (294)
6
57 La Lanthanum 138.91
58 Ce Cerium 140.12
59 Pr Praseodymium 140.91
60 Nd Neodymium 144.24
61 Pm Promethium (145)
62 Sm Samarium 150.36
63 Eu Europium 151.96
64 Gd Gadolinium 157.25
65 Tb Terbium 158.93
66 Dy Dysprosium 162.50
67 Ho Holmium 164.93
68 Er Erbium 167.26
69 Tm Thulium 168.93
70 Yb Ytterbium 173.05
71 Lu Lutetium 174.97
7
89 Ac Actinium (227)
90 Th Thorium 232.04
91 Pa Protactinium 231.04
92 U Uranium 238.03
93 Np Neptunium (237)
94 Pu Plutonium (244)
95 Am Americium (243)
96 Cm Curium (247)
97 Bk Berkelium (247)
98 Cf Californium (251)
99 Es Einsteinium (252)
100 Fm Fermium (257)
101 Md Mendelevium (258)
102 No Nobelium (259)
103 Lr Lawrencium (266)
For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
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